

*OPTIMALITY AND CONCURRENT VARIABLE-INTERVAL
VARIABLE-RATIO SCHEDULES*

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Despite claims to the contrary, all leading theories about operant choice may be seen as models of optimality. Although melioration is often contrasted with global maximization, both make the same core assumptions as other versions of optimality theory, including momentary maximizing, hill climbing, and the various versions of optimal foraging theory. The present experiment aimed to test melioration against more global optimality and to apply the visit-by-visit analysis suggested by foraging theory. Rats were exposed to concurrent schedules in which one alternative was always variable-ratio 10 and the other alternative was a variable-interval schedule. Although choice relations varied from rat to rat, the overall results roughly confirmed the matching law, a result often taken to support melioration. Pooling the data across sessions and across rats, however, resulted in no increment in unsystematic variance, lending support to the contention by Zirriax and Silberberg (1984) that the choice relation is partly constrained. When the data were analyzed at the level of visits, the results either disconfirmed predictions of melioration or showed regularities about which melioration is silent. Instead, performance tended toward a rough optimization, in which responding favored the variable ratio, but with relatively brief visits to the variable interval. There were no asymmetries in travel or variability that would indicate that different processes were involved in generating visits at the two different schedules. The findings point toward a more global optimality model than melioration and demonstrate the value of per-visit analysis in the study of concurrent performances.

Key words: concurrent schedules, concurrent variable interval variable ratio, optimality, foraging theory, per-visit analysis, lever press, rats

The matching law, discovered by Herrnstein (1961) in a study of concurrent pairs of variable-interval (VI) schedules, may be written

$$\frac{B_1}{B_2} = \frac{r_1}{r_2}, \quad (1)$$

where B_1 and B_2 represent behavior allocated to Alternatives 1 and 2, and r_1 and r_2 represent reinforcement obtained from Alternatives 1 and 2 as a result of B_1 and B_2 .

Since its discovery, the matching law has been tested in a wide variety of situations, for example with different species, responses, and reinforcers (Baum, 1979; Wearden & Burgess, 1982). Deviations from Equation 1 have been well captured by an extension that adds two parameters, producing an equation

that has come to be called the generalized matching law:

$$\log \frac{B_1}{B_2} = s \log \frac{r_1}{r_2} + \log b, \quad (2)$$

where s represents sensitivity of the behavior ratio, B_1/B_2 , to variation in the reinforcer ratio, r_1/r_2 , and b represents bias due to factors other than r_1 and r_2 in favor of one or the other alternative (Baum, 1974b, 1979; Davison & McCarthy, 1988).

Although Equation 2 has been successful in describing choice in a great variety of experiments, it remains a tool of description. Several attempts have been made to explain its ubiquity by reference to more basic principles of behavior (see Williams, 1988, for an overview). For example, Herrnstein and Vaughan (1980) proposed the principle of melioration. This theory stems from the assumption that behavioral allocation depends directly on the difference between the two alternatives' payoff ratios (r/B , where B may be measured either as number of responses or time spent responding; Baum & Rachlin, 1969):

$$D = \frac{r_1}{B_1} - \frac{r_2}{B_2}. \quad (3)$$

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If D is positive, behavioral allocation moves toward Alternative 1; if it is negative, allocation moves toward Alternative 2. With VI schedules, the movement of behavior toward an alternative lowers its payoff ratio, because r is limited by the schedule. The result is that behavioral allocation always corrects D back toward zero, equating the payoff ratios and (with a little algebraic rearranging) producing matching (Equation 1).

Although melioration is often presented and tested in contrast with global optimization of overall rate of reinforcement (so-called maximization; Herrnstein, 1990; Mazur, 1981; Vaughan, 1981; Vaughan & Herrnstein, 1987), this contrast is more correctly viewed as distinguishing two models of optimality. Optimal performance may be defined as the one that produces the best outcome. Although the best outcome may be defined as the highest level of overall reinforcement ($r_1 + r_2$), different versions of optimality theory make different assumptions about what is best, with the result that there is no one optimality theory, but rather many different models of optimality, depending on the assumptions (Stephens & Krebs, 1986). Of these assumptions, two are key: (a) the variable that defines *best* (often referred to as the currency of the model), and (b) out of all the dimensions of behavior that might affect the currency, the particular dimension that operates on the currency to allow it to attain the optimal level. At the core of all optimality models is the assumption that the effective dimension of behavior tends toward that level that produces the optimal level of the currency. Maximization and melioration share this core in common, but, as different versions of optimality theory, entail different assumptions about currency and behavior. For maximization, maximum overall reinforcement is best, and behavioral allocation (i.e., choice) moves toward the level at which overall reinforcement is maximized. For melioration, the highest payoff ratio is best, and behavior, measured as responses or time spent responding, moves toward the highest payoff ratio.

To make specific predictions, optimality models require more detailed assumptions. In particular, they must specify how one should calculate the currency. Discussions of maximization and melioration tend to be

vague on this point. One is usually left to decide whether to calculate overall reinforcement or payoff ratio over several sessions, one session, or some smaller time period. Without such specification, it is hard to distinguish a theory like melioration from the result it is supposed to predict (i.e., matching), because any deviation from the correct prediction may be attributed to incorrect calculation. In one of the few discussions of this problem, Mazur and Vaughan (1987) suggested that payoff ratio be calculated over several (four to six) reinforcers. When the method of calculation is specified like this, melioration comes to seem more like other short-term optimality models, such as hill climbing, which also focuses on switching based on local rates of reinforcement (Hinson & Staddon, 1983). If, using a window like this, one recalculated payoff ratio on every response, melioration would resemble momentary maximizing, in which each response is assumed to go to the alternative with the highest momentary probability of reinforcement (Shimp, 1969; Staddon, 1980; Staddon & Horner, 1989).

Specific testable predictions also require a second, complementary, specification: how to calculate the unit of behavior. Granted that the dimension of behavior that affects the currency is relative responding or rate of responding, over what period should this variable be calculated? Should one calculate it over several sessions, over a single session, over part of a session, over several reinforcers, from one reinforcer to the next? Studies in which patterns of choice are reinforced raise this problem when one is deciding how to change the rate of reinforcement in relation to behavioral allocation: The program running the experiment must calculate relative responding over some period of time or number of responses (e.g., Davison & Kerr, 1989; Heyman & Tanz, 1995).

One possible answer to this question of calculation window, at least in concurrent performance, is to measure behavior on each visit to an alternative. Because concurrent performance consists of responding at one alternative followed by a switch to another, the visit, defined as behavior from one switch to the next, presents an unambiguous level of measurement intermediate between individual responses and whole sessions.

Per-visit analysis is also useful because it co-

incides with the line of thought about operant choice experiments that likens them to foraging in nature. In this view, the situation represents an environment in which a resource is patchily distributed, with each alternative a resource patch (e.g., Davison, 1992; Gray, 1994; Shettleworth, 1988). Foraging theory focuses on visits to patches as the behavioral units of interest (Stephens & Krebs, 1986). Applying this idea to concurrent schedules, Houston and McNamara (1981) developed an optimality model in which visit duration varied so as to maximize overall reinforcement (see also Houston, McNamara, & Sumida, 1987). The model predicts deviations from matching as the rates of reinforcement at the two alternatives diverge, because the optimal strategy minimizes the duration of visits to the leaner alternative. Particularly as cost of switching between alternatives (travel) increases, the model predicts that visits to the richer alternative should be long, interrupted only occasionally by brief visits to the leaner alternative. Such a tendency would result in the sensitivity parameter in Equation 2 exceeding 1.0, an occurrence termed *overmatching* (Baum, 1979; Davison & McCarthy, 1988).

When these predictions were tested by varying travel and measuring visit durations, the pattern of brief visits to the lean side was found to emerge with increasing travel, and the predicted overmatching also occurred (Baum, 1982; Boelens & Kop, 1983). The model is further bolstered by the observation that when the commonly used changeover delay (COD; a period of nonreinforcement following a switch of alternatives) is treated as travel and thus is excluded from the calculation of visit duration, even typical concurrent pairs of VI schedules display overmatching (Baum, 1974a; Silberberg & Fantino, 1970).

Although Baum (1982) and Davison (1991) attempted to reconcile travel effects with matching by modifying Equation 2 so as to include travel as a punishment of switching, neither succeeded. Subtracting a punishment parameter from reinforcement at each alternative (Baum, 1982) failed to eliminate the overmatching, and treating the punishment as reinforcement lost during travel failed to incorporate the crucial instances in which the visits to the leaner alternative were briefer than the duration of travel, because

in those conditions the reinforcement at the leaner alternative would assume a negative value, presumably predicting that no switching ought to occur (Davison, 1991).

The present study brought the comparison of concurrent schedules to foraging together with a situation that has been taken as a test between melioration and maximization: concurrent pairs in which one schedule is a VI and the other is a variable ratio (VR). In such a pair, the VR alternative supplies a fixed payoff ratio because the probability of reinforcement for each response is the same (assuming the VR is programmed as a random ratio, in which each response has the same probability of reinforcement). The payoff ratio for the VI alternative, in contrast, can vary widely because many responses may be made for each reinforcer, or, if visits to the VI are infrequent enough, every response there can produce a reinforcer. Melioration (Equation 3) predicts that the payoff ratio for the VI will adjust to equal the probability of reinforcement for the VR, resulting in matching. Maximization of overall reinforcement predicts that most behavior should be allocated to the VR, with occasional visits to the VI. This pattern of visits may result in matching with a strong bias (b in Equation 2) toward the VR, undermatching (sensitivity less than 1.0), or overmatching, depending on the particular schedule pairs included in the experiment (Baum, 1981).

Some experimental results seemed to favor melioration because they showed matching, and although there was a bias toward the VR, the bias appeared to be smaller than maximization would require (Herrnstein & Heyman, 1979; Heyman & Herrnstein, 1986). Supporters of maximization responded by pointing to factors that might reduce the bias toward the VR, such as the value of activities other than responding at the two choice alternatives, such as "leisure" (Green, Rachlin, & Hanson, 1983; Rachlin, 1978; Rachlin, Green, & Tormey, 1988). Other experimental tests suggest that increased experience (Silberberg, Thomas, & Berendzen, 1991), the use of warmth as a reinforcer (Sakagami, Hursh, Christensen, & Silberberg, 1989), and improved stimulus control (Heyman & Tanz, 1995; Shurtleff & Silberberg, 1990) produce results more in keeping with maximization.

Complicating this picture further, Zirrax

and Silberberg (1984) argued that performance on concurrent VI VR, measured in the typical way of taking ratios of responses and reinforcers over several sessions (Equations 1 and 2), is constrained toward matching. Arguing from algebraic analysis, computer simulations, and data from pigeons, they contended that, unlike concurrent VI VI, in which the behavior ratio may vary widely with no effect on the reinforcer ratio, concurrent VI VR forces the reinforcer ratio to follow the response ratio. They found that, for a constant pair of schedules (e.g., concurrent VI 30 s VR 30) as the response ratio varied widely, the reinforcer ratio varied with it. They concluded that response ratio tends to adjust reinforcer ratio, rather than to adjust to it. They noted, however, that despite such a constraint, deviations from matching could occur if responding at the VI dropped to an extremely low level. In the terms of the foraging analogue, matching might appear as an artifact in concurrent VI VR when visits are aggregated, but choice might escape from the constraint if visits to the VI were infrequent and brief enough.

The present study aimed to test these theoretical approaches—melioration, maximization, and the foraging analogue—and to gain more detailed information about performance on concurrent VI VR. Melioration (Equation 3) was tested using several VI-VR pairs with the VR always set to 10 (probability of reinforcement of .1), travel was varied, and the data were recorded visit by visit.

METHOD

Subjects

The subjects were 6 adult male rats, bred at the University of New Hampshire from Long-Evans stock. They were housed individually in a climate-controlled vivarium with a 12:12 hr light/dark cycle, in which the dark period began between 10:00 and 11:00 a.m. Sessions were usually conducted around the switch to the dark phase, from 8:00 to 11:00 a.m. The rats were maintained throughout the experiment at 80% of free-feeding weights (determined when the rats were about 120 days old). They were fed laboratory chow each day following the day's sessions. Water was constantly available in the

home cages. The rats were 100 days old at the beginning of the experiment. Toward the end of the experiment, 3 rats died of respiratory illness, and 1 died as a result of breaking its teeth on the bars of its cage.

Apparatus

A rectangular box (147 cm long, 51 cm wide, and 19 cm high) was divided by wire mesh along most of its length to make a U-shaped space. Two response levers were mounted on the front wall, projecting 4 cm into the box, 3 cm from the floor, and 33 cm apart. Each lever required a force equivalent to a weight of 60 g moving a distance of 2 mm to operate a microswitch behind the wall. Two pellet hoppers were attached to the same wall, 33 cm apart and 7.5 cm to the right of each lever. Each pellet hopper was connected to a pellet dispenser. Reinforcers consisted of 45-mg grain-based pellets (Noyes Formula A). A microprocessor designed for process control (BCC-52 from Micromint, Inc.), programmed in the Basic language, monitored and controlled events in the apparatus. The box could be lit inside with six 28-V lights, three along each side wall.

A 23-cm section of the partition between the two levers could be removed to allow the rats to move directly (i.e., 33 cm) from one lever to the other. This condition was called *short travel*. With this section in place, passage from one lever to the other required traveling around the partition at 130 cm from the front wall, a total distance of at least 260 cm. This condition was called *long travel*.

Procedure

Schedules were always presented in pairs, a VR 10 associated with one lever and a VI schedule associated with the other. The VR 10 schedule was programmed by setting a .1 probability of reinforcement for each lever press (i.e., it was a random-ratio schedule). The VI schedule was programmed by randomly selecting intervals from 0 s to twice the average, each differing from the next by 5 ms. Table 1 shows the schedule pairs and the order in which they were studied. For each pair, three conditions were run: short travel, long travel, and then short travel again, except for the last pair, VR 10 VI 30 s, for which there was no return to short travel. There were 11 conditions (seven short travel and four long

Table 1

The schedule pairs studied. Each pair of schedules was studied with short travel, then long travel, and then short travel again, except for the last. The number of sessions was either 14 or 30. The rats for which the data met the criteria of stable performance are listed for each condition; a rat's number in parentheses indicates exclusive preference for one alternative.

Left lever	Right lever	Travel	Sessions	Rats
VR 10	VI 20 s	Short	30	102, 111, 120, 202, 210, 213
VR 10	VI 20 s	Long	30	102, 202, 210, 213
VR 10	VI 20 s	Short	14	111, 210, 213
VI 20 s	VR 10	Short	30	102, 111, 202, 210
VI 20 s	VR 10	Long	30	102, 111, (120), (202), (210), 213
VI 20 s	VR 10	Short	14	102, 120, (202), (210), 213
VI 30 s	VR 10	Short	30	102, 111, (120), 202, 210
VI 30 s	VR 10	Long	30	102, (120), 202
VI 30 s	VR 10	Short	30	102, 111, 120, 202
VR 10	VI 30 s	Short	30	
VR 10	VI 30 s	Long	30	102, 120

travel) in all. All conditions were conducted for 30 sessions, except for the returns to short travel following long travel with VI 20 s, which were conducted for 14 sessions.

At the start of a session, the lights along the side walls came on; at the end, they went off. A session ended after 60 reinforcers had been delivered or after 45 visits had occurred to each alternative, whichever came first. With rare exceptions, sessions were conducted daily.

RESULTS

For those rats that became ill, the data analysis included only those conditions up to the one in which the rat's health failed. Response ratios and reinforcer ratios were calculated for each session from the totals for the session, and the logarithms of these ratios were used to assess stability of performance. A rat's performance in a condition was considered stable if no new highest or lowest ratio for the condition occurred in the last 5 days of the condition. In order to insure that all performances analyzed were stable, any performance for any rat that failed to meet this criterion was excluded from the analysis. Table 1 lists, for each condition, the rats whose data were included in the analysis. For the standard analysis, response and reinforcer ratios were calculated from the totals for each session. The per-visit analyses relied on data recorded for each visit to a lever: the number of presses and reinforcers in the visit, and the times of the first and last presses in

the visit. Travel time was calculated by subtracting the time of the last press on one lever from the time of the first press on the other after a changeover. All per-visit measures were calculated by pooling individual visits to a lever across the five sessions. This allowed calculation of measures of central tendency and variability.

Figure 1 shows, in logarithmic coordinates, daily response ratios plotted against daily reinforcer ratios, rat by rat. Seven conditions, indicated in Table 1, were excluded from analysis because choice was exclusive in three or more sessions out of five: three conditions for Rat 120 favoring the VR, two of which were long-travel conditions, and two each for Rats 202 and 210 favoring the VI, one of which was a long-travel condition and the second of which was the short-travel condition following that one.

A striking feature of Figure 1 is that most of the triangles lie below the matching line and almost all of the circles lie above it. Whenever there was bias, it favored the VR, regardless of which side the VR was on. If lines were fitted to the triangles and circles separately, they would be similar, but opposite in bias.

Figure 2 shows the ratios of Figure 1 replotted with responses or reinforcers for the VR in the numerator, regardless of the positions of the VR and VI. The slopes of the fitted lines varied from 0.88 to 1.33, with a geometric mean of 1.09. The fits were generally good, r^2 averaging .88. The poor fit for Rat 111 may be attributed to a narrow range of

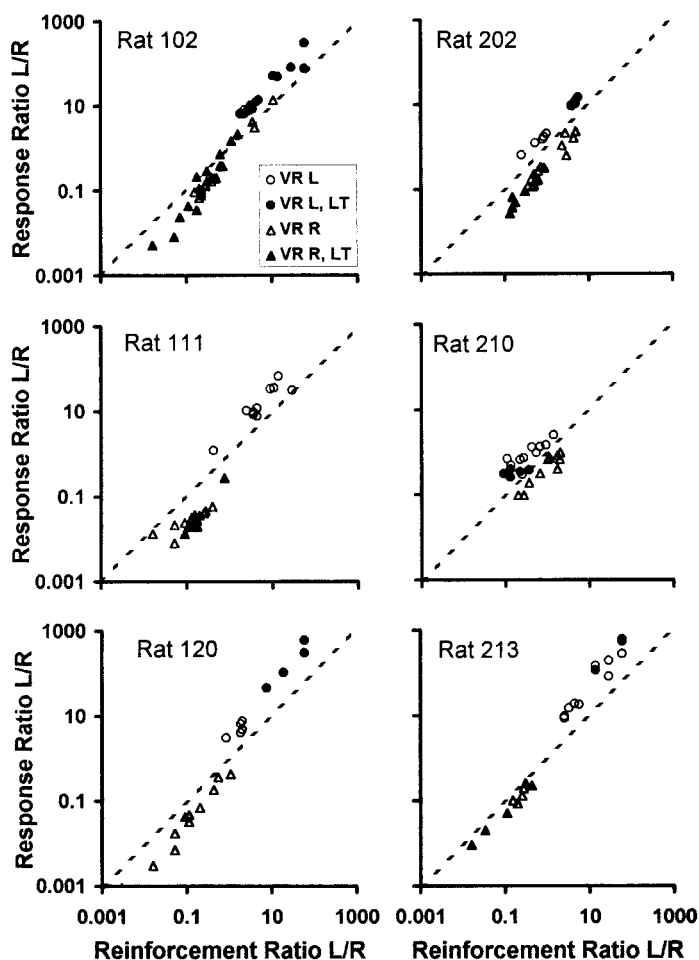


Fig. 1. Choice relations for individual rats. The ratio of presses per session on the left lever to presses per session on the right lever appears on the logarithmic y axis. The ratio of reinforcers per session on the left to reinforcers per session on the right appears on the logarithmic x axis. Each point represents one session, five points per condition (except for a few in which one or two sessions consisted of exclusive pressing on one lever). Broken lines indicate the locus of perfect matching. VR L: VR on the left (circles). VR R: VR on the right (triangles). LT: long travel (filled symbols). ST: short travel (open symbols).

variation in response and reinforcer ratios. The coefficients of the fitted equations all exceed 1.0, making the intercepts of all the lines positive, reflecting a bias in favor of the VR. Almost all points lie above the line of perfect matching. This finding confirms in rats the bias in favor of VR that was found previously for pigeons (e.g., Herrnstein & Heyman, 1979).

Table 2 compares the biases, sensitivities, and goodness of fit (r^2) calculated from medians of the daily ratios shown in Figure 2 (a common method of summarizing choice data) with those derived by calculating the

response and reinforcer ratios from the mean presses per visit (PPV) and mean reinforcers per visit (RPV) at the two alternatives. Sensitivities derived from the per-visit measures were either close to or greater than sensitivities derived by the standard method. The substantial undermatching for Rats 111 and 210 disappeared. The geometric mean of sensitivity for the per-visit analysis showed substantial overmatching. The bias in favor of the VR could be seen in both analyses, but disappeared for Rat 213 in the per-visit analysis. Goodness of fit was about the same for both analyses, except that the

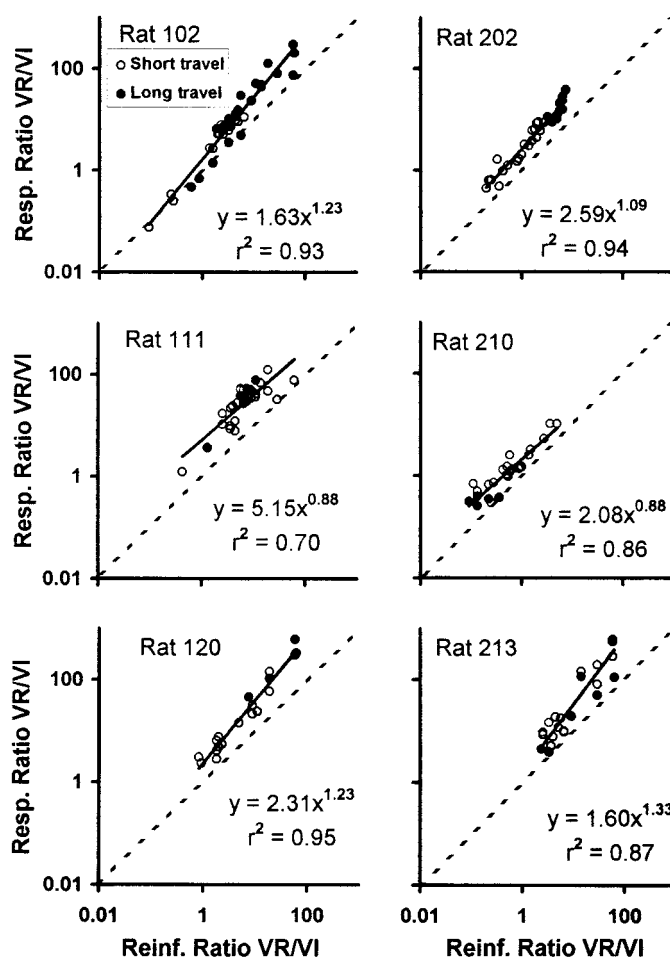


Fig. 2. Data of Figure 1 replotted as ratio of presses on the VR to presses on the VI versus ratio of reinforcers on the VR to reinforcers on the VI. Solid lines were fitted by the method of least squares. The equation of each regression line appears as a power function, the exponentiated version of Equation 2; the sensitivity, s , appears as the exponent, and the bias, b , appears as the coefficient. The proportion of variance accounted for (r^2) appears beneath each equation.

per-visit analysis produced a substantially better fit for Rat 111.

The graphs in Figure 2 vary in the range of response ratios and reinforcer ratios covered. For example, whereas Rat 213 always preferred the VR, Rat 210 showed only weak preference for the VR or preferred the VI. Although the lines fitted to the 2 rats' data resemble one another, the schedules permitted data points to lie in different ranges along the x axis. In contrast, performances on concurrent pairs of VI schedules always cover approximately the same range from subject to subject, because the obtained relative rein-

forcement approximates the programmed relative reinforcement (e.g., Baum, 1976). This variation in reinforcer ratio (x axis) with continued conformance to the matching relation made up part of the basis on which Ziriax and Silberberg (1984) argued that choice on concurrent VI VR is constrained toward matching.

Figure 3 shows all the data used in Table 2 pooled into two plots: one for 5-day median ratios and one for ratios of means across visits. Pooled this way, the data cover large ranges on both axes. The two regression lines are strikingly similar and reflect the results of Ta-

Table 2

Sensitivity, bias, and goodness of fit (r^2) for fits of Equation 2 to 5-day medians of daily ratios and to ratios of mean presses and reinforcers across all visits. Parameters are shown for each rat's data, and the bottom row shows geometric means across rats.

Rat	5-day medians			Per-visit means		
	Sensi- tivity	Bias	r^2	Sensi- tivity	Bias	r^2
102	1.23	1.59	.97	1.21	1.71	.95
111	.77	6.07	.56	1.01	3.93	.79
120	1.20	2.06	1.00	1.31	1.91	.97
202	1.17	2.18	.95	1.16	2.42	.96
210	.83	1.98	.97	.97	2.11	.90
213	1.44	1.40	.93	1.61	.91	.91
Geometric M	1.08	2.22	.88	1.19	1.98	.91

ble 2: overmatching on average (slope of 1.2) with an average bias in favor of the VR (a factor of about 2.2:1). Striking too are the high goodness of fits ($r^2 = .94$ and .95). Pooling across rats in no way lowered the adherence of the ratios to a single straight line. For concurrent pairs of VI schedules, in contrast, one expects that pooling across individuals will increase unsystematic variance because of variation in the individuals' sensitivities and biases (e.g., Baum, 1976).

Analyses of visit duration in seconds (from first press to last press in a visit) were much less systematic. Plots (not shown) comparable to those in Figure 2, using either daily duration ratio or ratio of mean durations as the measure of choice, showed more variability both across and within subjects. This lower reliability of the time measure probably arose from irregular pausing between lever presses.

Because counting presses produced more reliable results, visit duration was measured by number of presses for the rest of the analyses. In what follows, number of presses per visit will be referred to as *visit duration*.

In contrast with results for pigeons, in which travel requirements led to overmatching (Baum, 1982; Bolens & Kop, 1983), Figures 1, 2, and 3 show that the long travel requirement had little effect on choice in this experiment. There was only a weak tendency for the filled symbols to be more extreme. Even when more extreme, however, the points for long travel appear to deviate no more from the fitted lines than the points for short travel (open symbols). When lines were fitted to the points in Figure 3 for short and long travel separately, the slopes (1.20) were identical for 5-day medians but differed in the expected direction (1.19 vs. 1.28) for visit means. The difference, however, was insignificant in comparison with differences found in pigeons (Baum, 1982; Boelens & Kop, 1983).

Because the probability of reinforcement on the VR was held fixed at .1 throughout the experiment, melioration theory predicts that the probability of reinforcement on the VI should have remained invariant across conditions. To assess this prediction, Figure 4 shows the payoff ratio, r/B in Equation 3, plotted as a function of visit duration (presses per visit). A similar pattern appears for every rat. The line for the VR remains flat, near 0.1, as expected, but the line for the VI slopes upward to the left, indicating that the payoff ratio increased across conditions as visit duration decreased. In the strictest version of

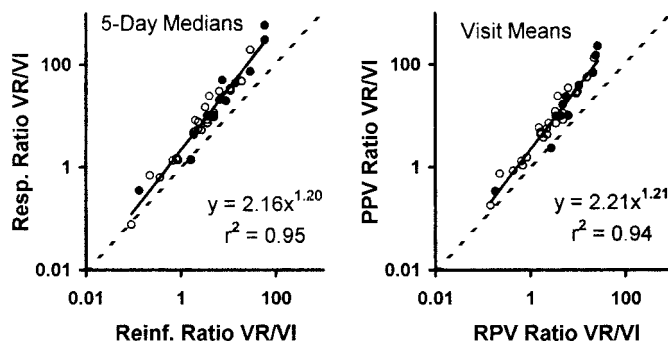


Fig. 3. Data of all 6 rats plotted together. The left graph shows the data from Figure 2 represented as 5-day medians. The right graph shows response and reinforcer ratios calculated from the mean presses per visit (PPV) and the mean reinforcers per visit (RPV). All conventions as in Figure 2.

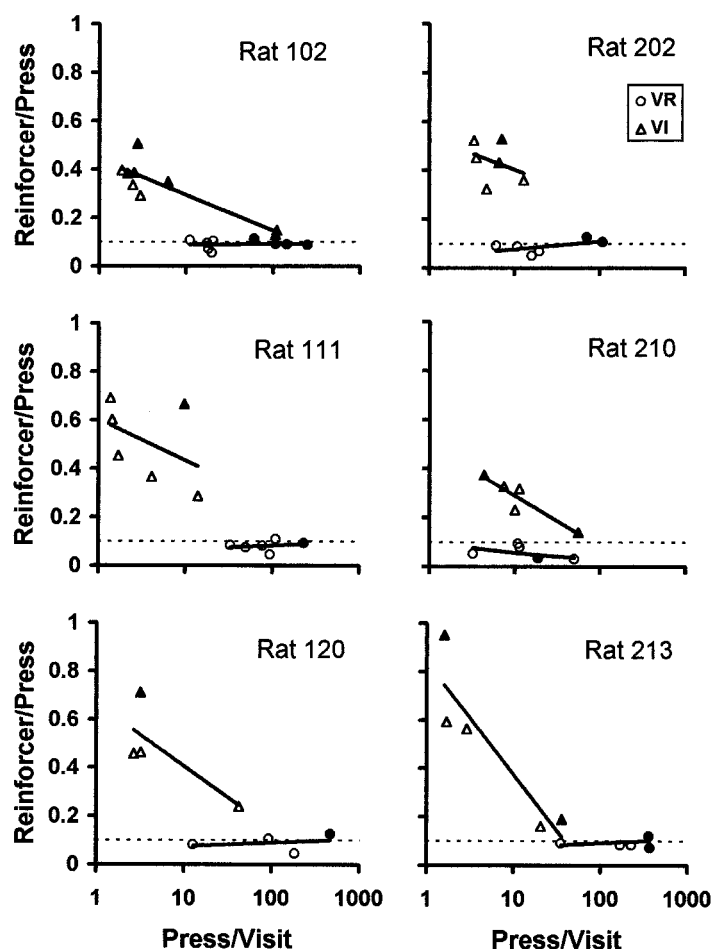


Fig. 4. Payoff ratio as a function of visit duration, both calculated as means of the per-visit data. Filled symbols indicate conditions with long travel. Broken horizontal lines indicate the payoff ratio of the VR, 0.1. Heavier lines (one for VR and one for VI) were fitted by the method of least squares. Note logarithmic x axis.

melioration (Equation 3), the two lines ought to coincide. If one allowed for bias toward the VR (Figures 2 and 3), they could separate, but melioration predicts that both lines should be flat. Instead, the sloping lines for the VI in Figure 4 suggest that as visits to the VI became brief they also became less frequent, with the result that they more often included reinforcement for one press or a few presses.

Figure 5 verifies that as visits to the VI became less frequent they also became shorter. It shows mean visit duration (PPV) plotted against preference, calculated as the ratio of the mean visit durations. For all 6 rats, as preference for the VR increased, VI visit duration decreased. The decrease, however, was

nonuniform; beyond a certain preference for the VR, the VI visits reached a minimum and no longer decreased, with the result that the shorter visits were all about equally brief. For all rats, the minimum average visit was fewer than 10 presses, and for 3 rats (Rats 102, 111, and 213), it approached one press. Figure 5 adds to the pattern inferred from Figure 4 that as VI visits became infrequent and payoff ratio increased, the visits also reached a minimum duration.

Figure 6 compares travel time to the VI with travel time to the VR by plotting one against the other. Points close to the major diagonal indicate equality. Long travel tended to take longer than short travel, but only for Rats 102 and 202 did the difference approach

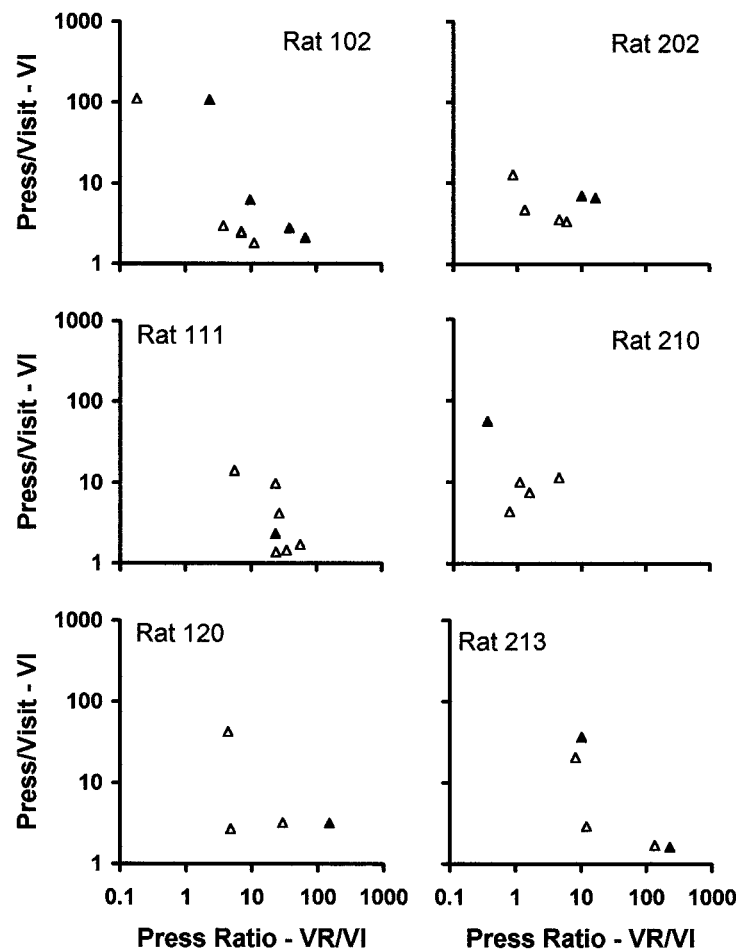


Fig. 5. Mean visit duration as a function of choice calculated as the ratio of mean visit durations, for VI. Filled symbols indicate conditions with long travel. Lines were fitted by the method of least squares. Note logarithmic axes.

the roughly 8:1 difference in distance. This convergence could occur because of pauses before, during, or after travel—pauses between the last press on one alternative and the first press on the other. Deviations from the major diagonal followed idiosyncratic patterns across rats. For Rat 102, travel to the VI appeared to vary inversely with travel to the VR for the short-travel conditions, but no other rat showed such a pattern. For Rat 202, travel time to the VR always exceeded that to the VI, but no other rat showed this pattern. For Rat 111, there was less variability in travel time to the VR than to the VI, but for Rat 210, the opposite was true. Although travel times often differed between schedules, they did so in no systematic way.

Although Figures 2 and 3 show a systematic bias in favor of the VR, Figure 6 indicates no systematic difference in travel between visits to the VI and visits to the VR. Even though travel revealed no difference, if the visits to the two schedules were qualitatively different or were generated by different processes, that might be reflected in different patterns of variability in visit duration. Figure 7 assessed this possibility. It shows, for each schedule in each condition, the standard deviation (*SD*) of visit duration (*PPV*) plotted against the mean. The broken line in each graph shows the locus of equality. For every rat, the variability not only covaried with the mean, but the standard deviation was always roughly equal to the mean. Both VR and VI and short

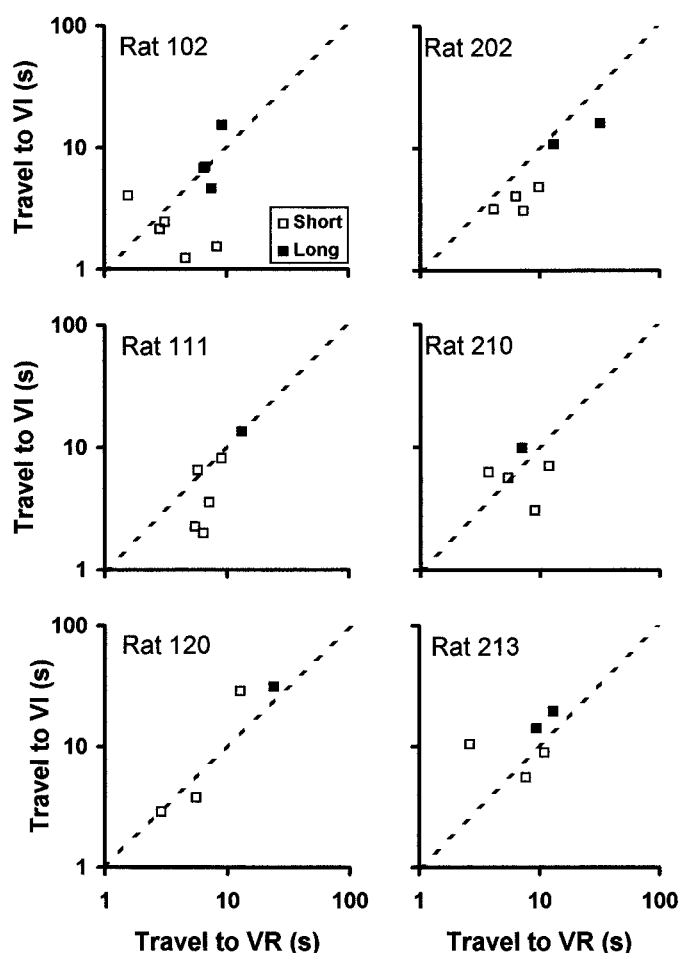


Fig. 6. Mean duration of travel from the VR to the VI (y axis) compared with mean duration of travel from the VI to the VR (x axis). Filled symbols indicate conditions with long travel. The broken line indicates the locus of equality. Note logarithmic axes.

travel and long travel appeared to conform to this pattern. The uniformity suggests that all visits were generated by a single process.

DISCUSSION

The results confirm some previous findings and contradict some previous theorizing. Although the analysis of choice in Figures 2 and 3 agrees with results found with pigeons (e.g., Heyman & Herrnstein, 1986), the analysis of visits goes against melioration and suggests instead an approximation to the performance predicted by maximization (e.g., Houston & McNamara, 1981).

Figures 2 and 3 show results similar to those found with pigeons: bias in favor of the

VR and approximate conformance with matching (Equations 1 and 2). Comparison of Figures 1 and 2 indicates that bias systematically favored the VR, regardless of whether it was on the left or right, and overwhelmed any bias toward one or the other position. Such a bias agrees with global maximization (Baum, 1981), but it may be reconciled with melioration by assuming a higher local response rate (or "tempo"; Baum & Rachlin, 1969) at the VR, with the result that the proportionality between number of responses and time spent responding assumed in Equation 3 would differ from VR to VI.

How we should interpret the apparent conformance with matching remains in doubt, however, because Figures 1, 2, and 3 support

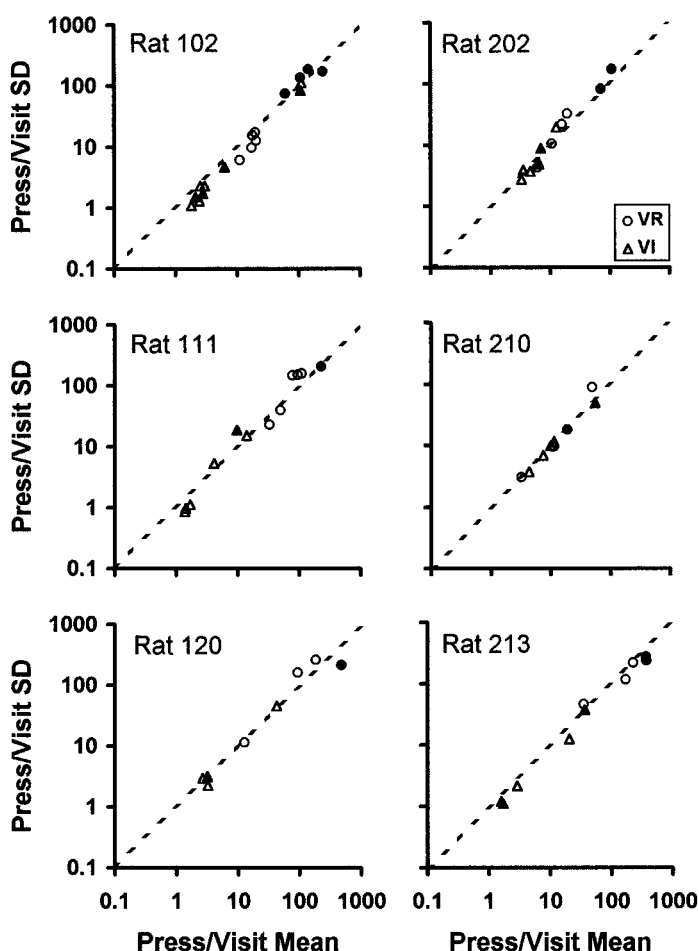


Fig. 7. Standard deviation (SD) of visit duration versus mean visit duration. Broken line indicates locus of equality. Filled symbols indicate conditions with long travel. Note logarithmic axes.

the contention of Ziriax and Silberberg (1984) that choice relations for concurrent VI VR are at least partly constrained toward an approximation of matching. For example, comparison of the fitted lines in Figure 2 with those in Table 2 indicates that plotting the results of all 5 days instead of a summary over the 5 days for each condition left bias, sensitivity, and goodness of fit almost unchanged. This could occur only if variation from day to day tended to coincide with variation across conditions. This coincidence of variation supports Ziriax and Silberberg's idea that the behavior ratio forces the reinforcer ratio toward matching it. Figure 3 shows another supportive uniformity: Regardless of variation in range of preference across rats, when the ratios of all rats were pooled, the conformance

to Equation 2 declined not at all. For the very same schedule, choice and reinforcer ratio varied widely from rat to rat, and yet matching was preserved. This suggests that the reinforcer ratio tended to vary with the behavior ratio. In the words of Ziriax and Silberberg, "Different animals on the same concurrent schedules had different choice ratios; yet, no matter what those ratios were, they matched the obtained reinforcement ratios" (p. 96). On this basis, they concluded, "Rather than choice ratios matching reinforcement ratios, reinforcement ratios match choice ratios" (p. 96), meaning that, because choice ratios partly determine the reinforcement ratios, the choice relation for concurrent VI VR is partly an artifact.

Although Figure 3 supports the argument

of Ziriaux and Silberberg (1984), it also contains a result that may contradict it: the significant overmatching indicated by the sensitivity of 1.2. Table 2 indicates that, for the analysis of visits, the tendency toward overmatching was evident across individual rats; only 2 rats approximately matched, whereas the other 4 showed clear overmatching. This result across rats and in the pooled data (Figure 3) suggests that the choice analysis is constrained only in part; that is, it can deviate significantly from matching. Even this is uncertain, however, because Ziriaux and Silberberg, using simulations, showed that the relation between response and reinforcer ratios for concurrent VI 30 VR 30 (i.e., the feedback function relating the ratios; Baum, 1973, 1981) was a line with a slope greater than 1.0 for moderate to high changeover rates. Although the present experiment included more than one pair of schedules, in aggregate they still might determine an overmatching relation. One conclusion is clear: The overmatching goes against melioration and supports a tendency toward maximization because Equation 3 predicts only matching, whereas more global versions of optimality predict the observed overmatching (Baum, 1982; Houston & McNamara, 1981).

Regardless of variations in details, any global maximization theory predicts that choice should favor the VR because behavior allocated to the VR pays off proportionately, whereas all reinforcement available at the VI may be obtained with occasional brief visits. Exceptions would occur only when the VR paid off at such a low rate that it was too costly to respond there or that responding there could only lower the rate obtained by responding on the VI; one would then expect exclusive responding at the VI. As long as the VR can pay off at rates comparable to or higher than the VI, performance should tend toward allocation of behavior mainly to the VR, with brief visits to the VI. Figures 4 and 5 indicate that such a tendency occurred in the present experiment.

Figure 4 shows a direct contradiction of melioration. Melioration, as embodied in Equation 3, predicts that the payoff ratio for the VI should equal or, allowing for bias, parallel the payoff ratio for the VR; the visit durations should always be long enough to allow the matching of payoff ratios. Figure 4 shows

that although payoff ratio for the VR remained approximately constant, for all 6 rats the payoff ratio for the VI increased as visits became briefer. Although it contradicts melioration, such a tendency toward strong preference for the VR coupled with brief visits to the VI suggests performances roughly in keeping with more global optimality (e.g., Houston & McNamara, 1981).

In contrast with results for pigeons (Baum, 1982; Boelens & Kop, 1983), increasing the length of travel between alternatives had little effect on the rats' behavior in the present study. Figure 6 shows that, although the long travel requirement usually generated longer travel times, the increase was far from proportional to the difference between short and long travel, and none of the other results suggests a strong effect, except for the preponderance of long-travel conditions that produced exclusive preference for one alternative. In contrast, research on travel with pigeons produced clear and strong effects (Baum, 1982). It may be that locomotion is more energetically costly for pigeons than for rats. There is a small amount of evidence that climbing travel for rats has large effects like travel for pigeons (Aparicio & Baum, 1997).

If the difference in schedules at the two alternatives were a key factor in producing the relation between VI payoff ratio and visit duration (Figure 4), one would expect that visits to the VR compared with visits to the VI would reveal some sort of asymmetry. Figure 6 reveals no obvious asymmetry in travel. The rats showed idiosyncratic differences between travel to the two schedules, but no systematic difference was seen across rats. The comparison done in Figure 7 between standard deviations and means of the visit durations might have revealed an asymmetry; if visits to the two schedules had been governed by two different processes, variability might have been differently related to central tendency. On the contrary, however, Figure 7 suggests no such difference; for both types of visit, the standard deviation was approximately equal to the mean. On this basis, it seems improbable that visits to the VR were generated by some different process than visits to the VI.

Figure 7 also illustrates the value of per-visit analysis of concurrent performances, because calculating the variability in visit duration was

impossible without it. Because averaging individual visits produced the same results as the more usual summarizing of performance at the level of sessions (Figure 3), we are reassured that no distortion results from per-visit analysis. In fact, the improved choice relations in Table 2 suggest that calculating choice from per-visit mean responses actually increases reliability. Because additional analyses become possible (e.g., Figure 7), and because use of computers to run experiments has made it easy, we may conclude that per-visit recording and analysis should be the preferred method for studies of concurrent performance.

The present results cast doubt on melioration as an explanation of performance on concurrent schedules. It is possible that some modification to Equation 3 would allow a new version of melioration to account for the variation in payoff ratios, but it is unclear what that would be or how it could remain simple. Instead, the present results suggest a tendency toward the performance predicted by more global optimality models: most behavior allocated to the VR, with brief and infrequent visits to the VI.

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